Definition
In a CFG, node $a$ dominates $b$ if every path from the start node to $b$ passes through $a$. Node $a$ is a dominator of $b$.

Property
The dominance relation is a partial order.

Definition
Node $a$ strictly dominates $b$ if $a \neq b$ and $a$ dominates $b$. 
Theorem

IF $a$ and $b$ both dominate $c$,
THEN either $a$ dominates $b$ or $b$ dominates $a$. 
**Theorem**

IF $a$ and $b$ both dominate $c$, THEN either $a$ dominates $b$ or $b$ dominates $a$.

**Corollary**

Every node $n$ has at most one immediate dominator $\text{idom}(n)$ such that

- $\text{idom}(n) \neq n$
- $\text{idom}(n)$ dominates $n$, and
- $\text{idom}(n)$ does not dominate any other dominator of $n$. 
Dominator Example
Computing Dominators

As a dataflow analysis

1. Forwards
2. Lattice is \( \mathcal{P}(\text{Stmts}), \supseteq \)
3. ∩
4. \( \text{out}_\ell = \text{in}_\ell \cup \{\ell\} \)
5. start node value is \{\}
6. \( \bot = \{\text{all statements}\} \)

More efficient approaches
Lengauer-Tarjan: see Appel book section 19.2
## Computing Dominators

### As a dataflow analysis

1. **Forwards**
2. Lattice is \( (\mathcal{P}(\text{Stmts}), \supseteq) \)
3. \( \cap \)
4. \( \text{out}_\ell = \text{in}_\ell \cup \{\ell\} \)
5. Start node value is \( \{} \)
6. \( \bot = \{\text{all statements}\} \)

### More efficient approaches

- Lengauer-Tarjan: see Appel book section 19.2
- Cooper, Harvey, Kennedy:  
A node $w$ is in the **dominance frontier of $x$** if:

- $x$ does not strictly dominate $w$, and
- $x$ dominates a predecessor of $w$. 

**Definition**
DF_{local}(x): the successors of \( x \) not strictly dominated by \( x \).

DF_{up}(y): nodes in DF(y) not strictly dominated by idom(y).

DF(x) = DF_{local}(x) \cup \bigcup \{ y \mid \text{idom}(y) = x \} \; DF_{up}(y).
Algorithm DF(x):
1:   $S = \{\}$
2:   for all nodes $w \in \text{succ}(x)$ do
3:     if idom($w$) $\neq x$ then
4:       $S \cup = \{w\}$
5:     /* $S$ is now $DF_{\text{local}}(x)$ */
6:   for all nodes $y$ for which idom($y$) = $x$ do
7:     /* below we compute $DF_{\text{up}}(y)$ */
8:       for all nodes $w \in DF(y)$ do
9:         if $x$ does not dominate $w$ or $x = w$ then
10:            $S \cup = \{w\}$
11:   return $S$
Restatement of definition of DF

\( w \in DF(x) \) for every \( x \) that dominates a predecessor of \( w \), but does not strictly dominate \( w \).

Algorithm \texttt{Compute DFs(\()\):

1: for all nodes \( w \) do
2: \hspace{1em} for all \( p \in \text{preds}(w) \) do
3: \hspace{2em} \( x = p \)
4: \hspace{1em} while \( x \neq \text{idom}(w) \) do
5: \hspace{2em} \( DF(x) \cup = \{w\} \)
6: \hspace{1em} \( x = \text{idom}(x) \)