Part 1: Value numbering

1. (14 marks) Perform local value numbering on the following basic block:

```c
1 a = b + c
2 b = b + c
3 c = b + c
4 d = a + c
5 e = b + c
6 f = d + a
7 g = e + b
```

Show the table mapping each variable to a value number, and the table mapping each expression involving value numbers to a value number. Also, transform the code so that it consists of only the following kinds of instructions:

- arithmetic operations involving only value-number variables (e.g. `v1 = v2 + v3`)
- copies from the original variables to value-number variables and vice versa (e.g. `v1 = a`)

Part 2: Dominance, SSA form, Loops

Consider the following code in a Jimple-like intermediate representation:

```c
1 A: read a, b, c, d, e, f, g
2 B: a = b + c
3 C: b = f * g
4 E: c = d - a
5 if(a < c) goto G
6 F: e = e + b
7 goto C
8 G: if(b == f) goto I
9 H: f = 5 * d
10 if(f <= a) goto H
11 J: g = b + c
12 I: d = a - 6
13 if(c > d) goto E
14 D: if(d <= a) goto B
15 print a, b, c, d, e, f, g
```

Note: each question depends on the previous question. Check your work carefully after completing each question, before going on to the next one.
2. (3 marks) Identify the basic blocks, and draw a basic block graph.

3. (11 marks) Draw the dominator tree for your basic block graph.

4. (11 marks) Determine the dominance frontier for each node in the basic block graph.

5. (7 marks) For each variable in the program, use the iterated dominance frontier algorithm to determine which basic blocks require a $\phi$ instruction.

6. (7 marks) Draw the basic block graph again, this time including $\phi$ nodes at the nodes you identified in the previous question. For each variable, uniquely number its definitions. Number each use of a variable to indicate its unique reaching definition.

7. (12 marks) Identify the back edges in the graph. For each back edge, list the nodes in its natural loop.

8. (3 marks) Identify the strongly connected subgraph(s) in the graph. Is the graph reducible?