Partially Ordered Structures for Hazard Detection *

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Abstract

Although Boolean algebra constitutes the main mathematical model for digital circuits, other algebras are useful for detecting hazards, which are unwanted signals caused by stray delays. These algebras have the form $A = (S, +, *, ^-, 0, 1)$, where S is a set used to represent signal values, $+$ and $*$ are binary operations on S corresponding to OR and AND gates, respectively, $^-$ is a unary operation on S corresponding to an inverter, and 0 and 1 are the usual binary logic values. In the most general algebra, S is an infinite set, $+$ and $*$ are commutative and associative, and de Morgan's laws hold. Hence, this is a commutative de Morgan bisemigroup. The other algebras that have been used in the past have finite underlying sets; they can all be derived from the infinite algebra. In particular, the algebras with 5, 8, and 13 elements are de Morgan bisemilattices, and can be represented by two partial orders corresponding to $+$ and $*$. The 6-element algebra is a de Morgan algebra, described by a single partial order. Finally, the 3-element algebra is a special de Morgan algebra, called ternary algebra. Simple set-theoretic characterizations have been found for de Morgan bisemilattices, de Morgan algebras, and ternary algebras. We survey these algebras and their applications.

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History and Bibliography

1 Early work on hazards

An early treatment of hazards appeared in 1951 in a book by Keister, Ritchie and Washburn [21]. In 1957 Huffman [20] proposed informal definitions of static and dynamic hazards, and provided some characterizations of these hazards. A more formal treatment of hazards was given by McCluskey [23] in 1962, and further results were obtained by Unger [28] in 1969. These early works used the two-valued Boolean algebra.

2 Ternary algebras

Three-valued algebra was first used by Goto [17, 18] (1948). Other early works using three-valued algebra for hazard detection include the papers by Moisil [24] (1956), Muller [25] (1959), Roginskii [26] (1959), Yoeli and Rinon [29] (1964), and Eichelberger [13] (1965), who introduced an efficient two-pass ternary simulation algorithm.

The relation between the results of ternary simulation and those of binary analysis was first conjectured by Brzozowski and Yoeli in 1979 [12]. The conjecture was proved by Brzozowski and Seger in 1986 [9, 10, 11]. A more general version, which does not require that the initial state of the circuit be stable, was described in [27].

Ternary algebra is treated in considerable detail in the 1995 monograph by Brzozowski and Seger [11]. Gate circuits, as well as CMOS circuits are discussed.

3 Finite algebras with more that three elements

A recent survey of multivalued algebras for hazard detection was presented by Brzozowski, Ésik, and Iland [7]. An earlier survey by Hayes was published in 1986 [19].

A five-valued algebra was presented by Lewis [22] in 1972. A six-valued algebra was derived by Hayes [19] in 1986 for the detection of static hazards. An eight-valued algebra is implicit in the 1974 works of Breuer and Harrison [2], and Fantauzzi [15]. In 1986 Hayes [19] also constructed a thirteen-valued algebra.

For further details concerning these algebras, and also some unsuccessful algebras, see [7].

4 Infinite algebra and its quotients

In 2000 Brzozowski and Ésik [5, 6] introduced an infinite hazard algebra C, which is a commutative de Morgan bisemigroup. They generalized Eichelberger's ternary simulation to C. Moreover, they studied quotient algebras of C with respect to three congruences, and showed that each successful multivalued hazard algebra is isomorphic to a quotient algebra of C. Simulation in C or in one of its quotients is capable of counting signal changes in digital circuits, as well as hazards. Counting signal changes is important, because the energy consumption in a circuit is proportional to the number of signal changes in the circuit.

In 2001 Gheorghiu [16] showed that the results of binary analysis are covered by the results of simulation in C, in the sense that all the signal changes predicted by binary analysis are also present in the simulation results. She also showed the converse result for a restricted class of feedback-free circuits; the general case remains open. Also open is the characterization in terms of binary analysis of the results of simulation in the quotient algebras of C.

5 Characterizations of certain algebras

A simple set-theoretic characterization of finite ternary algebra was discovered by Brzozowski, Lou and Negulescu [8]. This result was generalized to infinite ternary algebra by Ésik [14]. Free ternary algebras were studied by Balbes [1].

The set-theoretic characterization of [8] was generalized by Brzozowski to de Morgan algebras [4], and to certain de Morgan bisemilattices [3].

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